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**Feldman et al.**

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(54) **SYSTEM AND METHOD FOR DETERMINING HYBRID-MANUFACTURING PROCESS PLANS FOR PRINTED CIRCUIT BOARDS BASED ON SATISFIABILITY MODULO DIFFERENCE LOGIC SOLVER**

(58) **Field of Classification Search**  
USPC ..... 716/101  
See application file for complete search history.

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(57) **ABSTRACT**

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One embodiment can provide a system for determining a hybrid-manufacturing process plan for manufacturing a printed circuit board (PCB). During operation, the system can obtain a set of hybrid-manufacturing constraints. The set of hybrid-manufacturing constraints can include a set of primitives, a set of atoms, and an atom end-state vector. A primitive can represent an additive or a subtractive manufacturing process corresponding to one or more atoms of the PCB. An atom can correspond to a unit of spatial volume of the PCB. The system can determine a plurality of feasible hybrid-manufacturing plans based on the set of hybrid-manufacturing constraints. Each feasible hybrid-manufacturing plan can represent an ordering of the set of primitives satisfying the atom end-state vector. The system can determine costs for manufacturing the PCB using the plurality of feasible hybrid-manufacturing plans. The system can determine, based on the costs, an optimized hybrid-manufacturing plan for manufacturing the PCB.

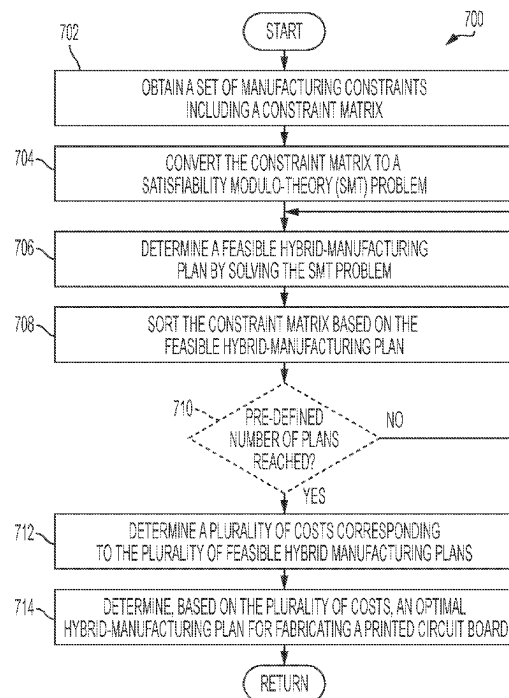
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**G06F 115/12** (2020.01)  
**G06F 7/24** (2006.01)  
**G06F 111/04** (2020.01)

(52) **U.S. Cl.**  
CPC ..... **G06F 30/337** (2020.01); **G06F 7/24** (2013.01); **G06F 2111/04** (2020.01); **G06F 2115/12** (2020.01)

**20 Claims, 8 Drawing Sheets**



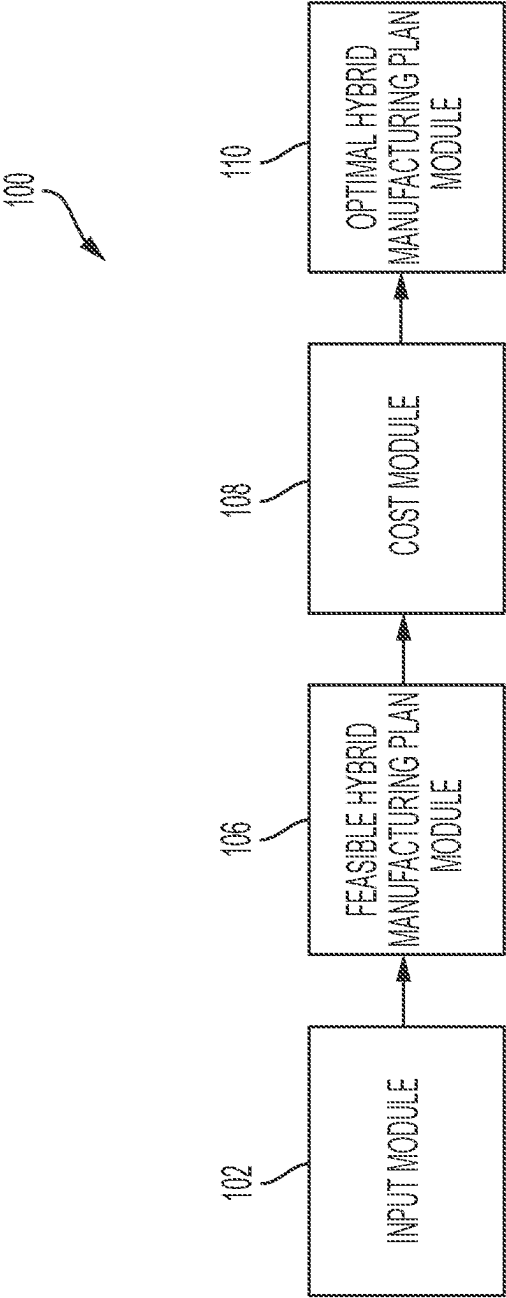


FIG. 1

200

Algorithm: CONVERTING CONSTRAINT MATRIX TO SMT

```
Input :  $M = \langle P, A, X, F, C \rangle$ , manufacturing constraints  
Output :  $\varphi$  SMT formula, conjunction of disjunction of inequalities  
 $\varphi \leftarrow \emptyset$   
for  $i \in 1, 2, \dots, m$  do  
   $\varphi' \leftarrow \emptyset$   
  for  $j \in 1, 2, \dots, n$  do  
    for  $k \in j+1, j+2, \dots, n$  do  
      if  $|x_j| = 1 \wedge x_k = 0$  then  
         $\varphi' \leftarrow \varphi' \vee x_k < x_j$   
      end  
    end  
  end  
   $\varphi \leftarrow \varphi \wedge \varphi'$   
end  
return  $\varphi$ 
```

FIG. 2

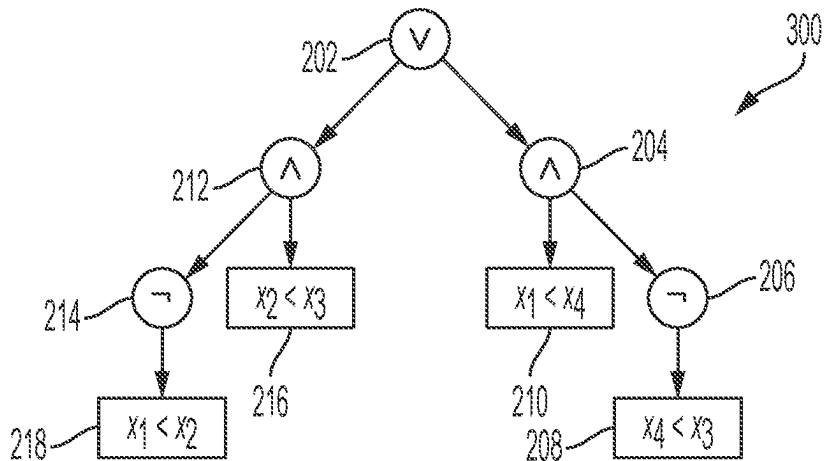


FIG. 3

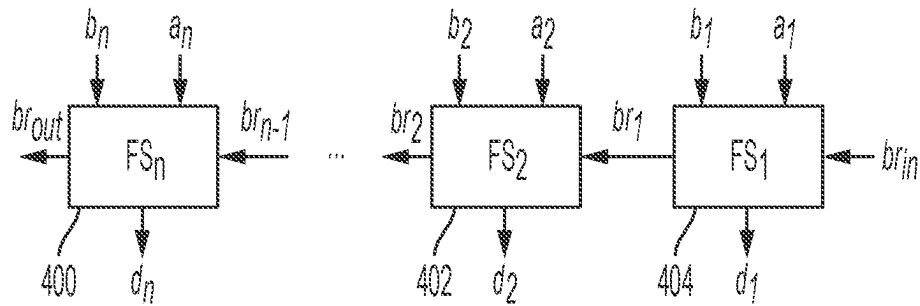


FIG. 4A

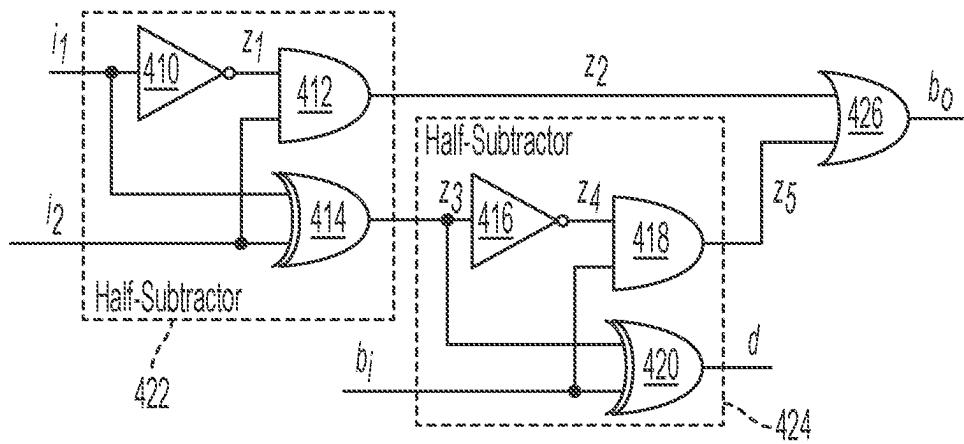


FIG. 4B

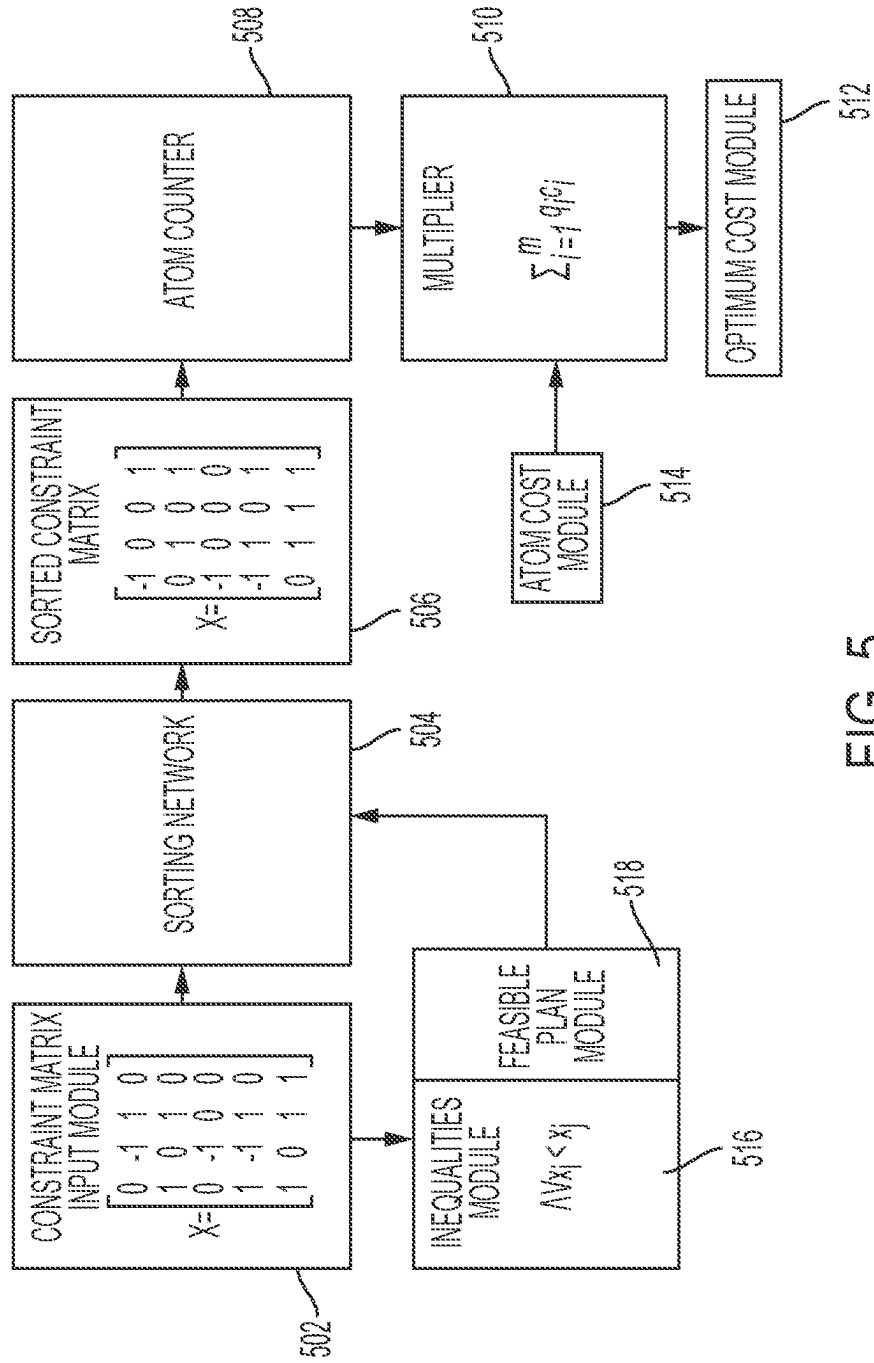


FIG. 5

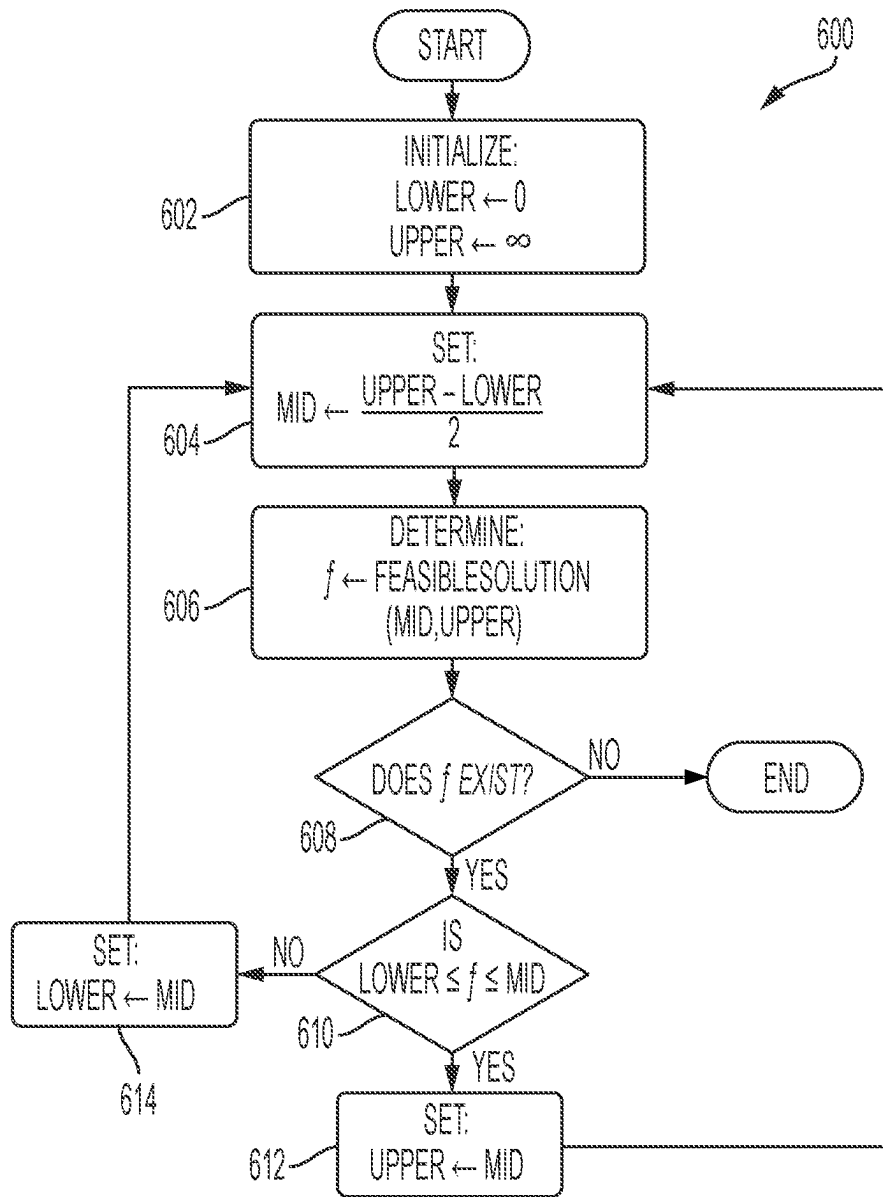


FIG. 6

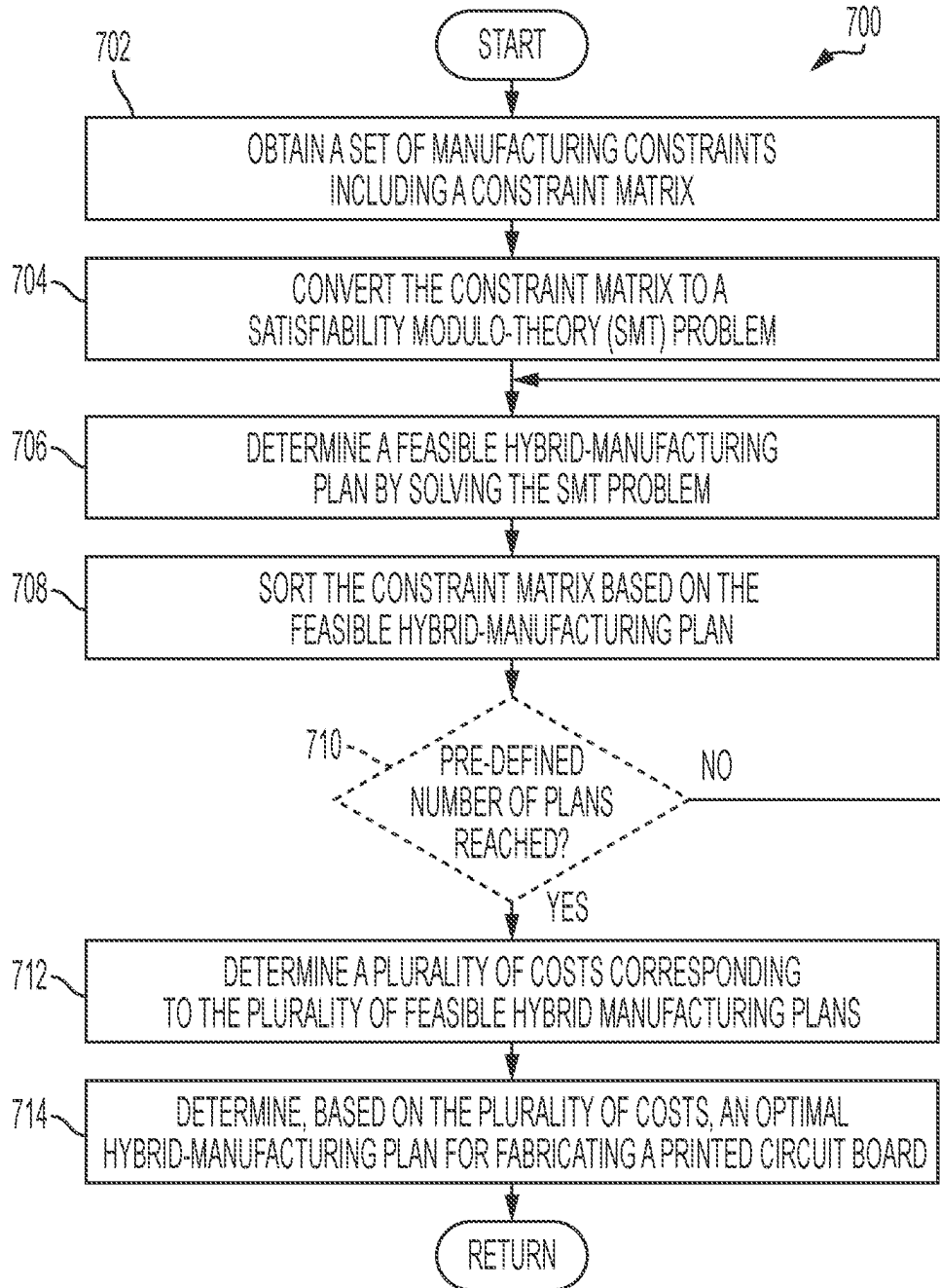


FIG. 7

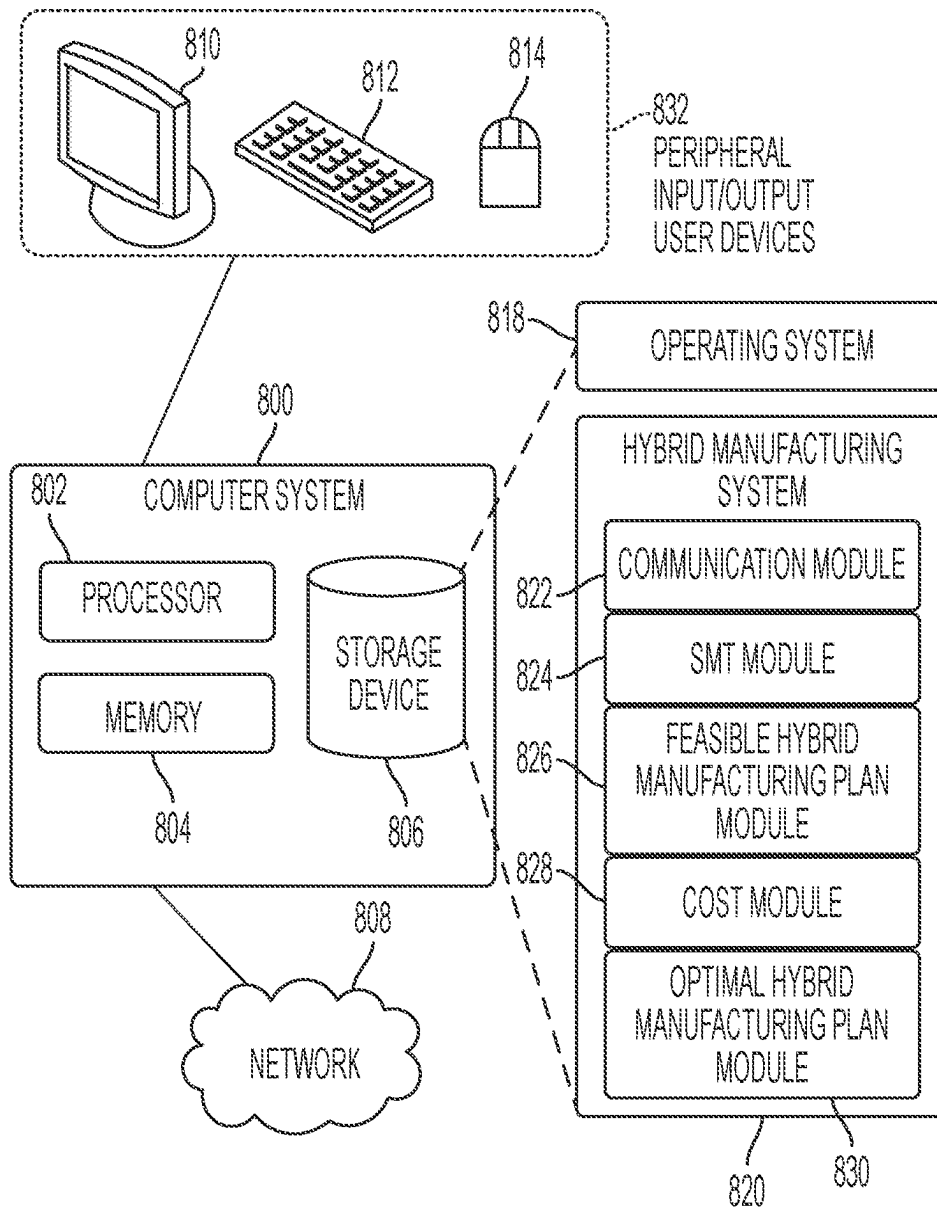


FIG. 8

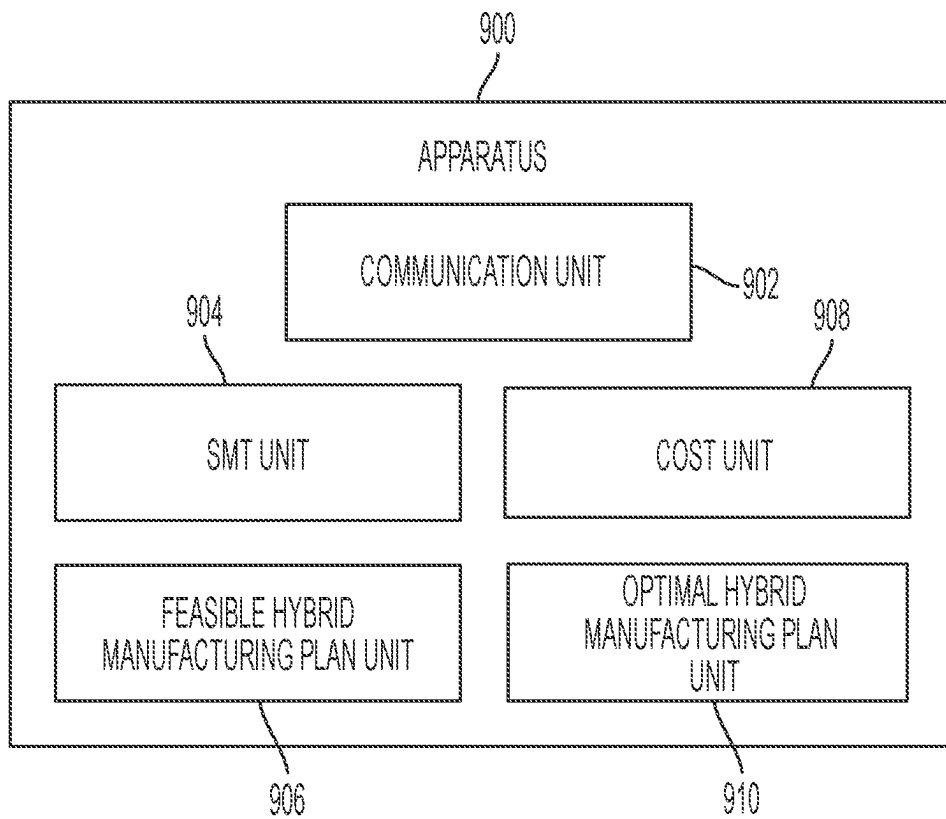


FIG. 9

**SYSTEM AND METHOD FOR  
DETERMINING HYBRID-MANUFACTURING  
PROCESS PLANS FOR PRINTED CIRCUIT  
BOARDS BASED ON SATISFIABILITY  
MODULO DIFFERENCE LOGIC SOLVER**

BACKGROUND

Field

This disclosure is generally related to hybrid-manufacturing planning. More specifically, this disclosure is related to a system and method for determining a hybrid-manufacturing process plan for printed circuit boards based on a satisfiability modulo difference logic solver.

Related Art

Computing manufacturing plans for pre-designed 3-dimensional (3D) objects is at the frontier in artificial intelligence (AI). Different types of manufacturing technologies are available for manufacturing complex 3D structures. Specifically, additive manufacturing technology can be used to manufacture complex 3D objects by adding materials in a layered fashion, e.g., adding material by a 3D printer. Subtractive manufacturing technology can also be used to manufacture 3D objects by removing material, e.g., by using a chemical bath. Some manufacturing technologies leverage the advantages of additive and subtractive manufacturing by combining the two for fabricating the 3D object. However, performing additive and subtractive manufacturing separately, e.g., by separate machines, may result in additional post-processing operations before transitioning from one manufacturing technology to another. Such additional post-processing can result in increased time-to-market and can also increase the cost of manufacturing the 3D object.

With the advancement in manufacturing technologies, new systems are capable of combining additive and subtractive manufacturing techniques in a single machine to perform a hybrid-manufacturing process. Current hybrid-manufacturing approaches typically first complete additive manufacturing steps followed by subtractive manufacturing steps. However, a hybrid-manufacturing process faces some challenges with respect to planning different manufacturing steps for fabricating the physical 3D object in a cost-effective and efficient way.

SUMMARY

According to one embodiment of the present invention, a system and method for determining a hybrid-manufacturing process plan for manufacturing a printed circuit board (PCB). During operation, the system can obtain a set of hybrid-manufacturing constraints for manufacturing the PCB. The set of hybrid-manufacturing constraints can include a set of primitives, a set of atoms, and an atom end-state vector. An atom can correspond to a unit of spatial volume of the PCB. A primitive can represent an additive or a subtractive manufacturing process corresponding to one or more atoms of the PCB. Next, the system can determine a plurality of feasible hybrid-manufacturing plans based on the set of hybrid-manufacturing constraints. Each feasible hybrid-manufacturing plan can represent an ordering of the set of primitives that satisfies the atom end-state vector. The system can then determine costs for manufacturing the PCB using the plurality feasible hybrid-manufacturing plans. The

system can determine, based on the costs, an optimized hybrid-manufacturing plan for manufacturing the PCB.

In a variation on this embodiment, the set of hybrid manufacturing constraints can include: a constraint matrix with the columns corresponding to the set of primitives and the rows corresponding to the set of atoms; and an atom cost vector.

In a further variation on this embodiment, the system can convert the set of hybrid-manufacturing constraints to a satisfiability modulo theory (SMT) problem. The system can determine the feasible hybrid-manufacturing plans by solving a Satisfiability (SAT) modulo difference logic.

In a further variation on this embodiment, each cost can be associated with one or more of: tool set-up cost; shut-down cost; and material cost.

In a variation on this embodiment, each manufacturing process can include one or more of: a plating process; an imaging process; a chemical bath process; and an etching process.

In a further variation, the system can convert the set of hybrid-manufacturing constraints to the SMT problem by: generating a conjunctive normal form (CNF) modulo difference logic formula; and converting the CNF difference logic formula to a Boolean formula, wherein the Boolean formula is implemented by using at least one or more Boolean subtractor circuits.

In a variation of this embodiment, the system can determine the plurality of feasible hybrid-manufacturing plans by performing for a respective feasible hybrid-manufacturing plan the following operations: sorting one or more columns in a constraint matrix based on one or more variables in the feasible hybrid-manufacturing plan, wherein each column corresponds to a primitive, and wherein sorting the one or more columns in the constraint matrix corresponds to changing an order of the set of primitives determining, for each primitive, an atom count representing a number of atoms added or deleted; determining an atom cost associated with each primitive; and determining a cost of the feasible hybrid-manufacturing plan by multiplying the atom cost and the atom count for respective primitives and aggregating across the set of primitives.

In a variation of this embodiment, the system can determine, based on the plurality of costs, the optimized hybrid-manufacturing plan by applying a binary search to the plurality of costs to determine the optimized hybrid-manufacturing plan.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 shows an exemplary system block diagram for determining a hybrid-manufacturing plan, in accordance with one embodiment of the present disclosure.

FIG. 2 shows an exemplary algorithm for converting a constraint matrix to a SMT formula, in accordance with one embodiment of the present disclosure.

FIG. 3 shows an example SMT function, in accordance with one embodiment of the present disclosure.

FIG. 4A shows an exemplary ripple-borrow subtractor circuit, in accordance with one embodiment of the present disclosure.

FIG. 4B shows an exemplary example of a full-subtractor circuit, in accordance with one embodiment of the present disclosure.

FIG. 5 illustrates an exemplary block diagram of a system performing a SMT augmented function, in accordance with one embodiment of the present disclosure.

FIG. 6 presents a flowchart illustrating a process for finding a cost-optimal solution, in accordance with one embodiment of the present disclosure

FIG. 7 presents a flowchart illustrating a process for determining a hybrid manufacturing plan by using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure.

FIG. 8 illustrates an exemplary computer system that facilitates a hybrid-manufacturing planner using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure.

FIG. 9 illustrates an exemplary apparatus that facilitates a hybrid-manufacturing planner using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure.

In the figures, like reference numerals refer to the same figure elements.

### DETAILED DESCRIPTION

The following description is presented to enable any person skilled in the art to make and use the embodiments, and is provided in the context of a particular application and its requirements. Various modifications to the disclosed embodiments will be readily apparent to those skilled in the art, and the general principles defined herein may be applied to other embodiments and applications without departing from the spirit and scope of the present disclosure. Thus, the present invention is not limited to the embodiments shown, but is to be accorded the widest scope consistent with the principles and features disclosed herein.

#### Overview

Embodiments described herein solve the technical problem of determining optimized hybrid-manufacturing plans for manufacturing a PCB. Specifically, a system can determine a sequence of additive or subtractive operations in a cost-effective and efficient way to obtain the object. During operation, the system can obtain a set of hybrid-manufacturing constraints that can include a final state vector and a constraint matrix including a set of primitives and a set of atoms, which can correspond to different spatial volumes associated with the PCB. That is, the 3D space in which the pre-designed PCB is embedded can be partitioned into a number of spatial volumes, with each volume represented by an atom. A subcollection of these atoms form the pre-designed PCB, or an approximation of it as deemed sufficiently accurate by user-specified tolerances.

Each primitive in the set of primitives can add or remove a subcollection of atoms representing the spatial volume in a single additive or subtractive manufacturing operation, respectively. The atoms represent the smallest spatial volumes that can be added or removed at once, however, they cannot be added or removed independently. They are physically constrained to be added or removed alongside other atoms in a primitive representing a feasible manufacturing action. Each primitive is computed by analyzing the geometry of the pre-designed PCB part against the tool shape, machine degrees of freedom, and possibly other manufacturing parameters. The atoms are subsequently computed as canonical intersection regions among primitives and their complements in 3D space.

A hybrid-manufacturing process plan (or simply a “plan”) is defined by a sequence of manufacturing actions represented by adding or removing the primitives. A plan is feasible if it produces the pre-designed PCB, or an approximation of it as deemed sufficiently accurate by user-specified tolerances. To achieve this end-goal, it is necessary and

sufficient to have every atom inside the object present and every atom outside the object absent at the end of a process plan. The sufficient conditions for this to happen can be specified by a constraint matrix obtained by analyzing how the order of primitives in the sequence affects the presence or absence of atoms at the end of the plan. Each feasible plan can represent a different ordering of the primitives for manufacturing the same PCB, i.e., the same subcollection of atoms that end up being present after the different sequences of additive and subtractive primitives are applied. Each of the feasible plans may result in manufacturing the pre-designed PCB at a different cost, which normally depends on how the atoms appear and disappear in the intermediate stages of the plan. Hybrid-manufacturing process planning may refer to finding one or a plurality of feasible plans, all feasible process plans, or any distinguished subset of feasible process plans; for instance, the most cost-effective process plans

The system can convert the constraint matrix to a satisfiability modulo theory (SMT) problem that can be solved to determine a plurality of feasible hybrid-manufacturing plans. To determine a feasible hybrid-manufacturing plan that is cost effective and efficient, the system may integrate a cost module to compute a cost of each plan. The output of the cost module can be a plurality of costs corresponding to the plurality of feasible hybrid-manufacturing plans. Based on the plurality of costs, the system can apply a search technique to determine an optimized hybrid-manufacturing plan for manufacturing the pre-designed PCB.

#### System and Method for a Hybrid-Manufacturing Planner

FIG. 1 shows an exemplary system block diagram for determining a hybrid-manufacturing plan, in accordance with one embodiment of the present disclosure. Typically, a PCB can include a number of parts, e.g., a substrate or base which is conventionally made of fiberglass; a copper portion for carrying electrical signals; a solder mask portion or a polymer portion that is usually applied on top of the PCB to protect copper traces in the copper layer from corrosion and short circuits; and/or a silkscreen portion for identifying PCB components that would be added.

To accommodate the increase in the complexity of electronic circuits and the density of circuit components, a multi-layer PCB are currently being used. Manufacturing each PCB layer that matches the design specifications with user-specified design tolerances can involve a number of manufacturing processes. Each manufacturing process can represent an action. A hybrid-manufacturing plan associated with the PCB can correspond to a sequence of actions such as imaging, etching, copper deposition, plating, layering and drilling, applying solder mask, chemical bath, silk-screening, etc. These actions can be categorized as either additive or subtractive processes for manufacturing the PCB. In hybrid-manufacturing, it is desirable to carefully plan the ordering of the additive and subtractive manufacturing processes so that the final fabricated PCB matches the design specifications in a cost-effective way.

In the example shown in FIG. 1, system 100 can determine a hybrid-manufacturing plan for a pre-designed PCB. System 100 can include an input module 102 that can define a set of manufacturing constraints. Specifically, a hybrid-manufacturing problem M can be defined as a tuple (P, A, X, F, C) that includes multiple sets of constraints defined as:

$$P = \{p_1, p_2, \dots, p_m\} \quad (1)$$

$$A = \{a_1, a_2, \dots, a_n\} \quad (2)$$

$$X \in \{-1, 0, 1\}^{m \times n} \quad (3)$$

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$$F \in \{-1, 0, 1\}^n \quad (4)$$

$$C \in \mathbb{Z}^m \quad (5)$$

where P denotes a set of primitives, A denotes a set of atoms, X represents a constraint matrix, F represents an atom end-state vector, and C denotes an atom cost vector.

Primitives can be grouped into two categories, additive manufacturing (AM) primitives and subtractive manufacturing (SM) primitives. An AM primitive or a SM primitive can characterize a manufacturing step or a manufacturing capability (e.g., imaging, etching, copper deposition, plating, layering and drilling, applying solder mask, chemical bath, silk-screening, etc). For the purpose of defining the hybrid-manufacturing constraints, the pre-designed PCB can be divided into volume units (each of which can have a different shape) and each unit of volume can be represented as an atom. Specifically, an atom can represent a spatial volume in Euclidean space that is classified as completely inside or completely outside against all primitives i.e., the one or more primitives that include that atom will add or remove them upon the additive or subtractive primitive's action, respectively.

The constraint matrix X (shown in equation (3)) is an n×m matrix with the rows representing atoms A and the columns representing primitives P denoted by equations (2) and (1), respectively. The set of manufacturing constraints may include additional constraints, e.g., co-ordinates in Euclidean space to indicate where to perform an etching operation, type of tool to be used to perform an operation, description of the toolset, type of material on which operation is to be performed, type of material to be added, temperature related constraints, etc. The constraint matrix X can be represented as an array of (-1, 0, 1) assignments with two bits per integer. Table 1 below describes the meaning of different values associated with element  $x_{i,j}$  in the constraint matrix X.

TABLE 1

Description of element values in constraint matrix X	
Constrain matrix X element values	Description
$x_{i,j} = 1$	primitive $p_j$ adds atom $a_i$
$x_{i,j} = -1$	primitive $p_j$ removes atom $a_i$
$x_{i,j} = 0$	primitive $p_j$ neither adds nor removes atom $a_i$

Atom end-state vector F represents the final design specification that is desired to be satisfied. An element in the atom end-state vector F can be denoted as f. Table 2 below describes the meaning of different values in the atom end-state vector F.

TABLE 2

Description of element values in atom end-state vector F	
Final state vector, F, values	Description
$f_i = 1$	atom $a_i$ desired to be filled in the final design
$f_i = -1$	atom $a_i$ is desired to be empty in the final design
$f_i = 0$	atom $a_i$ can be filled or empty in the final design

Table 2: Description of element values in atom end-state vector F

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For example, hybrid-manufacturing system **100** characterized by four primitives, i.e.,  $P = \{p_1, p_2, p_3, p_4\}$ , and five atoms, i.e.,  $A = \{a_1, a_2, a_3, a_4, a_5\}$ , can be associated with the following set of manufacturing constraints data:

$$X = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix}; F = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ -1 \end{bmatrix}; C = \begin{bmatrix} 10 \\ 2 \\ 2 \\ 90 \\ 8 \end{bmatrix} \quad (6)$$

Vector C in equation (6) denotes a cost vector corresponding to the manufacture of the five atoms by applying primitives defined in constraint matrix X.

System **100** can include a feasible hybrid-manufacturing plan module **102** to determine, based on the manufacturing constraints, a plurality of feasible hybrid-manufacturing plans. Given the hybrid-manufacturing constraints M, with primitives P (as denoted in equation (1)), a hybrid-manufacturing plan P(M) can represent a total ordering of the primitives in P. Feasible hybrid-manufacturing plan module **102** can compute a plan P(M) to provide a total ordering of the primitives in P that when implemented may result in a final state of atoms in A that can satisfy the atom end-state vector F. In other words, in each hybrid-manufacturing step, a primitive  $p_{j \in \{1, 2, \dots, m\}}$  may add an atom  $a_{i \in \{1, 2, \dots, n\}}$  if the atom is not present (and have no effect on it otherwise); or a primitive  $p_j$  may remove the atom  $a_i$  if the atom is present (and have no effect on it otherwise). At the last hybrid-manufacturing step, an atom is desired to be present when  $f_i = 1$  and an atom is desired to be absent when  $f_i = -1$ . Feasible hybrid-manufacturing plan module **102** may determine a number of such feasible hybrid-manufacturing plans, however all of them may not result in an optimal feasible hybrid-manufacturing plan that is cost effective and efficient.

Therefore, a cost module **108** is integrated into system **100** to determine a cost for each of the feasible hybrid-manufacturing plans. Specifically, given constraints M and a feasible hybrid-manufacturing plan P(M), cost module **108** can determine a cost of the plan, where the cost can be defined as:

$$\text{cost}(P) = \sum_{i=1}^m q_i c_i \quad (7)$$

where  $q_i \in \mathbb{Z}$  denotes the number of times an atom is added or removed by a feasible hybrid-manufacturing plan P(M) and  $c_i$  denotes the cost associated with each primitive when implementing plan P(M).

In response to cost module **108** determining a cost(P) for each feasible hybrid-manufacturing plan P(M), an optimal hybrid-manufacturing plan module **110** can determine an optimal hybrid-manufacturing plan such that the cost(P) is minimized. The operations of feasible hybrid-manufacturing plan module **106**, cost module **108**, optimal hybrid-manufacturing plan module **110** are further described in relation to FIGS. 2-7.

FIG. 2 shows an exemplary method for converting a constraint matrix to an SMT formula, in accordance with one embodiment of the present disclosure. Method **200** represents a planning process. In other words, method **200** can convert the problem of determining a feasible hybrid-

manufacturing plan and an optimal hybrid-manufacturing plan into an SMT problem. In one embodiment, the SMT can be difference logic theory, hence method 200 can convert a hybrid-manufacturing planning problem to a satisfiability (SAT) modulo difference logic.

Method 200 may convert the hybrid-manufacturing planning problem to a Conjunctive Normal Form (CNF) format, thus enabling the system to determine the satisfiability of a CNF formula using SAT modulo difference logic. A CNF modulo difference logic formula over a set of integer variables,  $X = \{x_1, x_2, \dots, x_n\}$  can be defined as a conjunction of disjunction of literals:

$$\varphi = \bigwedge_{p \in P} \bigvee_{q \in Q} p < q \tag{8}$$

where  $\{p, q\} \subseteq X^2$ . Given the manufacturing constraints the system can convert a constraint matrix defined in equation (3) to an SMT formula according to method 200. In other words, given the manufacturing constraints, method 200 can determine the precedence of the primitives. For example, a primitive can only add material to an atom if that material has not previously been added, and a primitive can only remove a material only if the material exists. Therefore, proper ordering of the primitives can be necessary for correctly manufacturing different complex structures in the pre-designed PCB.

FIG. 3 shows an example SMT function, in accordance with one embodiment of the present disclosure. The example SMT function can implement the formula determined by algorithm shown in FIG. 2. Specifically, given the constraint matrix X, the formula can be denoted in CNF as:

$$\varphi = (\neg(x_1 < x_2) \wedge (x_2 < x_3)) \vee (x_1 < x_4) \wedge (x_4 < x_3) \tag{9}$$

The Boolean operators used in equation (9) are negation ( $\neg$ ), disjunction ( $\vee$ ), and conjunction ( $\wedge$ ). In FIG. 3, the Boolean function  $\varphi$  defined in equation (9) can be represented in the form of a tree topology 300. The leaf nodes in tree 300 represent variables with inequalities, i.e., 208, 210, 216, and 218, and these leaf nodes with inequalities can be called as the theory. The variables  $\{x_1, x_2, \dots, x_n\}$  can be integer variables. The non-leaf nodes are Boolean operators 202-206, 212, and 214. For example, the negation operation 214 on inequality 218, i.e.,  $\neg(x_1 < x_2)$ , can result in inequality  $(x_1 \geq x_2)$ . The inequality  $(x_1 < x_2)$  can indicate that primitive  $x_1$  precedes primitive  $x_2$ .

At node 212 a conjunction operation is performed on 216 and the inequality  $(x_1 \geq x_2)$ , to generate an inequality  $(x_1 \geq x_2) \wedge (x_2 < x_3)$ . Similarly, operations 206 and 204 on inequalities 208 and 210, respectively, can result in  $(x_4 \geq x_3) \wedge (x_1 < x_4)$ . A disjunction operation 202 on the results of the two branches in tree 300, i.e.,  $(x_1 \geq x_2) \wedge (x_2 < x_3)$  and  $(x_4 \geq x_3) \wedge (x_1 < x_4)$  is denoted in equation (9). The inequality  $(x_2 < x_3)$  can indicate that primitive  $x_2$  precedes primitive  $x_3$ . Similarly, inequality  $(x_1 < x_4)$  can indicate that primitive  $x_1$  precedes primitive  $x_4$ .

In one embodiment, the system can determine a feasible hybrid-manufacturing plan from the SMT formula shown in equation (9) by implementing an SMT solver. The output of the SMT solver can correspond to an assignment of all variables  $x_i$  in  $\varphi$  such that  $\varphi$  evaluates to a Boolean constant T. Optionally, the system can convert  $\varphi$  to a Boolean formula by encoding every integer variable  $x_i$  as a Boolean vector and each proposition of type  $x_i < x_j$  can be replaced by a Boolean subtractor. A Boolean subtractor using ripple borrow architecture is described below in relation to FIGS. 4A and 4B.

FIG. 4A shows an exemplary ripple-borrow subtractor circuit, in accordance with one embodiment of the present disclosure. An n-bit ripple-borrow subtractor can include a

cascade of N full subtractors 400-404. A full subtractor is a combinational circuit that can have three inputs and two outputs. Specifically, each full subtractor (FS) in FIG. 4A can have three binary inputs and two binary outputs. For example, consider FS 402 that has three inputs, i.e.,  $a_2, b_2,$  and  $br_1$ . The binary inputs  $a_2$  and  $b_2$  represent the inputs that are to be subtracted, and input  $br_1$  can represent a borrow bit. If a subtraction operation in FS 404 associated with the least significant bit, resulted in a borrow operation borrow bit  $br_7$  will be set to 1 otherwise it will be set to zero. Output of subtraction operation performed by FS 402 is denoted as  $d_2$  and if a borrow operation was performed by FS 402 then  $br_2$  will be set to one otherwise it will be set to zero. Alternatively, instead of the ripple-borrow full subtractor the system can use borrow-look-ahead subtractors, two's complements, sign inverters, or full-adders.

FIG. 4B shows an exemplary example of a full-subtractor circuit, in accordance with one embodiment of the present disclosure. A full-subtractor circuit can subtract two binary numbers  $i_1$  and  $i_2$ . The full-subtractor circuit can include a third input, i.e., a borrow input bit  $b_i$ . The outputs can be the difference  $d$  and the borrow output  $b_o$ . The full-subtractor circuit can be implemented by using two half-subtractors, i.e., 422 and 424, and an OR gate 426. Each half-subtractor, i.e., 422 and 424, can be implemented by one XOR gate, one AND gate, and one NOT gate. Specifically, half subtractor 422 can be implemented by XOR gate 414, AND gate 412, and NOT gate 410. Likewise, half subtractor 424 can be implemented by XOR gate 420, AND gate 418, and NOT gate 416.

The system can solve the SMT formula shown in equation (9) to generate a feasible hybrid-manufacturing plan. In one embodiment, the system in response to finding the first feasible hybrid-manufacturing plan may apply different permutations of the primitives associated with the constraint matrix to generate a corresponding SMT formula. The system may solve each SMT formula using the SMT solver to generate a plurality of feasible hybrid-manufacturing plans. Alternatively, the system can also randomize the SMT solver to return a different solution each time the SMT solver is implemented. Applying a stochastic search over the plurality of feasible hybrid-manufacturing plans may return a feasible solution with a good cost.

To perform a search for a cost-optimal solution, the system may compute a cost associated with each feasible hybrid-manufacturing plan. Therefore, to determine a cost-optimal hybrid-manufacturing plan, the system may incorporate in the SMT formula the computation of the cost and may add a binary search space including costs associated with the plurality of feasible hybrid-manufacturing plans. An architecture of such an SMT augmented formula that can also compute costs is described below in relation to FIG. 5.

FIG. 5 illustrates an exemplary block diagram of a system performing a SMT augmented function, in accordance with one embodiment of the present disclosure. System 500 may include a constraint matrix input module 502 that provides a constraint matrix to inequalities module 516 where the constraint matrix can be converted to a SMT formula using the algorithm shown in FIG. 2. System 200 can then apply a feasible plan module 518 to determine a feasible plan. The system can use the variables in the feasible plan as keys in a sorting network 504. Sorting network 504 can sort the columns of the constraint matrix to generate a sorted constraint matrix shown in 506, each sorting can correspond to a different ordering of the primitives. An atom counter module 508 can compute the number of added or deleted atoms per primitive. System 500 can then apply a multiplier

**508** to multiply the output, i.e., atom count, of counter module **508** with integer atom costs according to equation (7). Atom cost module **514** can compute a cost associated with each atom. System **500** can then apply an optimum cost module **512** to determine a cost-optimal solution. Specifically, system **500** may apply optimal cost module **512** to perform a binary search over all the feasible solutions and can determine a feasible solution with an optimal cost.

FIG. **6** presents a flowchart illustrating a process for finding a cost-optimal solution, in accordance with one embodiment of the present disclosure. In one embodiment the system uses an SMT solver to determine a plurality of feasible hybrid-manufacturing plans, the system may further incorporate in the SMT solver a binary search over a search space including the plurality of feasible hybrid-manufacturing plans. First, the system may initialize a lower and upper bound in the search space (operation **602**). The system can then determine a median of the search space based on the lower and upper bounds (operation **604**). The system can determine the presence of an optimal-feasible solution based on the costs of respective feasible hybrid-manufacturing plans in the second half of the search space, i.e., from the median of the search space to the upper bound of the search space (operation **606**).

The system can determine whether a cost-optimal solution exists in the selected search space (operation **608**). If no solution exists then the operation ends, otherwise the system can continue to determine whether the cost-optimal feasible solution exists in the first half of the search space (operation **610**). If the condition in operation **610** is satisfied, then the system can update the upper bound to the median value (operation **612**) and the operation continues to label **604**. In other words, the system may identify a presence of a better solution in the first half of the search space, therefore the system may continue to narrow the search in the first half of the search space. If the condition in operation **610** is not satisfied, the system can update the lower bound to the median value (operation **614**) and the operation continues to label **604**. In other words, the system may identify a presence of a better solution in the second half of the search space, therefore the system may continue to narrow the search in the second half of the search space. The system can continue the search until a cost-optimal feasible hybrid-manufacturing plan is found. With the incorporation of the cost computation and the binary search operations, the SMT solver can find a specific ordering of primitives that results in a cost-optimal feasible hybrid-manufacturing plan.

FIG. **7** presents a flowchart illustrating a process for determining a hybrid manufacturing plan by using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure. During operation, the system can obtain a set of manufacturing constraints defined in equations (1)-(4) (operation **702**). The set of manufacturing constraints can include a constraint matrix which can provide a comprehensive description of the type of primitives and the atoms on which the primitives can operate. The system can then convert the constraint matrix to a SMT problem (operation **704**). Next, the system can determine a feasible hybrid-manufacturing plan by solving the SMT problem (operation **706**). The feasible hybrid-manufacturing plan provides a certain ordering of the primitives that satisfies the constraints in the atom end-state vector defined in equation (4). Based on the feasible hybrid-manufacturing plan, the system can sort the constraint matrix to generate a different constraint matrix (operation **708**).

The system may optionally determine whether a pre-defined number of feasible hybrid-manufacturing plans have been computed (operation **710**). When the condition in operation **710** is not satisfied, the system can continue to operation **706** to determine a different feasible hybrid-manufacturing plan in each iteration until the pre-defined number of feasible hybrid-manufacturing plans have been reached. When the condition in operation **710** is satisfied, the system can determine a plurality of costs corresponding to the plurality of feasible hybrid-manufacturing plans (operation **712**). The system can then determine, based on the plurality of costs, a cost-optimal hybrid manufacturing plan for manufacturing a PCB (operation **714**).

Exemplary Computer System and Apparatus

FIG. **8** illustrates an exemplary computer system that facilitates a hybrid-manufacturing planner using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure. In this example, computer system **800** can include a processor **802**, a memory **804**, and a storage device **806**. Computer system **800** can be coupled to peripheral input/output (I/O) user devices **830**, e.g., a display device **810**, a keyboard **812**, and a pointing device **814**, and can also be coupled via one or more network interfaces to network **808**. Storage device **806** can store instructions for an operating system **818** and a hybrid manufacturing system **820**.

In one embodiment, hybrid manufacturing system **820** can include instructions, which when executed by processor **802** can cause computer system **800** to perform methods and/or processes described in this disclosure. Hybrid manufacturing system **820** can include a communication module **822** to receive a set of manufacturing constraints. Hybrid manufacturing system **820** can further include instructions implementing a SMT module **824** for converting the constraint matrix to a SMT problem.

Hybrid manufacturing system **820** can include a feasible hybrid manufacturing plan module **826**, which can determine a feasible hybrid manufacturing plan by using an SMT solver. Feasible hybrid manufacturing plan module **826** can further sort the constraint matrix to generate a different constraint matrix. For each new constraint matrix, feasible hybrid manufacturing plan module **826** can determine a different feasible hybrid-manufacturing plan. Therefore, feasible hybrid manufacturing plan module **826** can iteratively generate a plurality of feasible hybrid-manufacturing plans.

Hybrid manufacturing system **820** can also include a cost module **828** for determining a plurality of costs for the corresponding plurality of feasible hybrid-manufacturing plans. Hybrid manufacturing system **820** can further include an optimal hybrid-manufacturing plan module **830** to determine a cost-optimal hybrid-manufacturing plan by for example applying a binary search to the search space including the plurality of feasible plans with associated costs. Hybrid manufacturing system **820** may then use communication module **822** to output the cost-optimal hybrid-manufacturing plan for manufacturing the PCB.

FIG. **9** illustrates an exemplary apparatus that facilitates a hybrid-manufacturing planner using a satisfiability modulo difference logic solver, in accordance with one embodiment of the present disclosure. Apparatus **900** can include units **902-910**, which perform functions or operations similar to modules **822-830** of computer system **800** in FIG. **8**, respectively. Apparatus **900** can include: a communication unit **902**, a SMT unit **904**, a feasible hybrid-manufacturing plan unit **906**, a cost unit **908**, and an optimal hybrid-manufacturing plan unit **910**.

The methods and processes described in the detailed description section can be embodied as code and/or data, which can be stored in a computer-readable storage medium as described above. When a computer system reads and executes the code and/or data stored on the computer-readable storage medium, the computer system performs the methods and processes embodied as data structures and code and stored within the computer-readable storage medium.

Furthermore, the methods and processes described above can be included in hardware modules or apparatus. The hardware modules or apparatus can include, but are not limited to, application-specific integrated circuit (ASIC) chips, field-programmable gate arrays (FPGAs), dedicated or shared processors that execute a particular software module or a piece of code at a particular time, and other programmable-logic devices now known or later developed. When the hardware modules or apparatus are activated, they perform the methods and processes included within them.

The foregoing descriptions of embodiments of the present invention have been presented for purposes of illustration and description only. They are not intended to be exhaustive or to limit the present invention to the forms disclosed. Accordingly, many modifications and variations will be apparent to practitioners skilled in the art. Additionally, the above disclosure is not intended to limit the present invention. The scope of the present invention is defined by the appended claims.

What is claimed is:

1. A method for determining a hybrid-manufacturing process plan for manufacturing a printed circuit board (PCB), the method comprising:

obtaining, by a computer, a set of hybrid-manufacturing constraints for manufacturing a PCB, wherein the set of hybrid-manufacturing constraints include a set of primitives, a set of atoms, and an atom end-state vector, wherein an atom corresponds to a unit of spatial volume of the PCB, and wherein a primitive represents an additive or subtractive manufacturing process corresponding to one or more atoms of the PCB;

determining a plurality of feasible hybrid-manufacturing plans based on the set of hybrid-manufacturing constraints, wherein each feasible hybrid-manufacturing plan represents an ordering of the set of primitives that satisfies the atom end-state vector;

determining costs for the manufacturing the PCB using plurality of feasible hybrid-manufacturing plans; and determining, based on the costs, an optimized hybrid-manufacturing plan for manufacturing the PCB.

2. The method of claim 1, wherein the set of hybrid manufacturing constraints further comprises:

a constraint matrix with the columns corresponding to the set of primitives and the rows corresponding to the set of atoms; and  
an atom cost vector.

3. The method of claim 1, further comprising converting the set of hybrid-manufacturing constraints to a satisfiability modulo theory (SMT) problem;

wherein determining the feasible hybrid-manufacturing plans comprises solving the SMT problem using a -Satisfiability (SAT) modulo difference logic.

4. The method of claim 1, wherein each cost is associated with one or more of:

tool set-up cost;  
shut-down cost; and  
material cost.

5. The method of claim 1, wherein each manufacturing process includes one or more of:

a plating process;  
an imaging process;  
a chemical bath process; and  
an etching process.

6. The method of claim 1, further comprising converting the set of hybrid-manufacturing constraints to a satisfiability modulo theory problem by:

generating a conjunctive normal form (CNF) modulo difference logic formula; and

converting the CNF difference logic formula to a Boolean formula, wherein the Boolean formula is implemented by using at least one or more Boolean subtractor circuits.

7. The method of claim 1, wherein determining the plurality of feasible hybrid-manufacturing plans further comprises:

for a respective feasible hybrid-manufacturing plan:

sorting one or more columns in a constraint matrix based on one or more variables in the feasible hybrid-manufacturing plan, wherein each column corresponds to a primitive, and wherein sorting the one or more columns in the constraint matrix corresponds to changing an order of the set of primitives; determining, for each primitive, an atom count representing a number of atoms added or deleted; determining an atom cost associated with each primitive; and

determining a cost of the feasible hybrid-manufacturing plan by multiplying the atom cost and the atom count for respective primitives and aggregating across the set of primitives.

8. The method of claim 1, wherein determining the optimized hybrid-manufacturing plan further comprises:

applying a binary search to the costs to determine the optimized hybrid-manufacturing plan.

9. A non-transitory computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for determining a hybrid-manufacturing process plan for manufacturing a printed circuit board (PCB), the method comprising:

obtaining a set of hybrid-manufacturing constraints for manufacturing the PCB, wherein the set of hybrid-manufacturing constraints include a set of primitives, a set of atoms, and an atom end-state vector, wherein an atom corresponds to a unit of spatial volume of the PCB, and wherein a primitive represents an additive or subtractive manufacturing process corresponding to one or more atoms of the PCB;

determining a plurality of feasible hybrid-manufacturing plans based on the set of hybrid-manufacturing constraints, wherein each feasible hybrid-manufacturing plan represents an ordering of the set of primitives that satisfies the atom end-state vector;

determining costs for manufacturing the PCB using the plurality of feasible hybrid-manufacturing plans; and determining, based on the costs, an optimized hybrid-manufacturing plan for manufacturing the PCB.

10. The non-transitory computer-readable storage medium of claim 9, wherein the set of hybrid manufacturing constraints further comprises:

a constraint matrix with the columns corresponding to the set of primitives and the rows corresponding to the set of atoms; and  
an atom cost vector;

wherein each manufacturing process includes one or more of:

a plating process;

a imaging process;  
 a chemical bath process; and  
 an etching process.

11. The non-transitory computer-readable storage medium of claim 9, wherein the method further comprises 5  
 converting the set of hybrid-manufacturing constraints to a satisfiability modulo theory (SMT) problem; and wherein determining the feasible hybrid-manufacturing plans comprises solving the SMT problem using a Satisfiability (SAT) modulo difference logic.

12. The non-transitory computer-readable storage medium of claim 9, wherein the method further comprises 10  
 converting the set of hybrid-manufacturing constraints to a satisfiability modulo theory problem by:

generating a conjunctive normal form (CNF) modulo 15  
 difference logic formula; and  
 converting the CNF difference logic formula to a Boolean formula, wherein the Boolean formula is implemented by using at least one or more Boolean subtractor circuits.

13. The non-transitory computer-readable storage medium of claim 9, wherein determining the plurality of 20  
 feasible hybrid-manufacturing plans further comprises:

for a respective feasible hybrid-manufacturing plan:  
 sorting one or more columns in a constraint matrix 25  
 based on one or more variables in the feasible hybrid-manufacturing plan, wherein each column corresponds to a primitive, and wherein sorting the one or more columns in the constraint matrix corresponds to changing an order of the set of primitives; 30  
 determining, for each primitive, an atom count representing a number of atoms added or deleted;  
 determining an atom cost associated with each primitive; and  
 determining a cost of the feasible hybrid-manufactur- 35  
 ing plan by multiplying the atom cost and the atom count for respective primitives and aggregating across the set of primitives.

14. The non-transitory computer-readable storage medium of claim 9, wherein determining the optimized 40  
 hybrid-manufacturing plan further comprises:

applying a binary search to the plurality of costs to determine the optimized hybrid-manufacturing plan.

15. A computer system for determining a hybrid-manu- 45  
 facturing process plan for manufacturing a printed circuit board (PCB), the system comprising:

a processor; and  
 a storage device coupled to the processor and storing 50  
 instructions, which when executed by the processor cause the processor to perform a method, wherein the method comprises:

obtaining a set of hybrid-manufacturing constraints for 55  
 manufacturing the PCB, wherein the set of hybrid-manufacturing constraints include a set of primitives, a set of atoms, and an atom end-state vector, wherein an atom corresponds to a unit of spatial volume of the PCB, and wherein a primitive represents an additive or subtractive manufacturing process corresponding to one or more atoms of the PCB;  
 determining a plurality of feasible hybrid-manufactur- 60  
 ing plans based on the set of hybrid-manufacturing

constraints, wherein each feasible hybrid-manufac-  
 turing plan represents an ordering of the set of  
 primitives that satisfies the atom end-state vector;  
 determining costs for manufacturing the PCB using the  
 plurality of feasible hybrid-manufacturing plans; and  
 determining, based on the costs, an optimized hybrid-  
 manufacturing plan for manufacturing the PCB.

16. The computer system of claim 15, wherein the set of 5  
 hybrid manufacturing constraints further comprises:

a constraint matrix with the columns corresponding to the  
 set of primitives and the rows corresponding to the set  
 of atoms; and  
 an atom cost vector;  
 wherein each manufacturing process includes one or more  
 of:  
 a plating process;  
 a masking process;  
 a chemical bath process; and  
 an etching process.

17. The computer system of claim 15, wherein the method 10  
 further comprises converting the set of hybrid-manufactur-  
 ing constraints to a satisfiability modulo theory (SMT)  
 problem; and

wherein determining the feasible hybrid-manufacturing  
 plans comprises solving the SMT problem using a  
 Satisfiability (SAT) modulo difference logic.

18. The computer system of claim 15, wherein the method 15  
 further comprises converting the set of hybrid-manufactur-  
 ing constraints to a satisfiability modulo theory problem by:  
 generating a conjunctive normal form (CNF) modulo  
 difference logic formula; and

converting the CNF difference logic formula to a Boolean  
 formula, wherein the Boolean formula is implemented  
 by using at least one or more Boolean subtractor  
 circuits.

19. The computer system of claim 15, wherein determin- 20  
 ing the plurality of feasible hybrid-manufacturing plans  
 further comprises:

for a respective feasible hybrid-manufacturing plan:  
 sorting one or more columns in a constraint matrix  
 based on one or more variables in the feasible  
 hybrid-manufacturing plan, wherein each column  
 corresponds to a primitive, and wherein sorting the  
 one or more columns in the constraint matrix corre-  
 sponds to changing an order of the set of primitives;  
 determining, for each primitive, an atom count repre-  
 senting a number of atoms added or deleted;  
 determining an atom cost associated with each primi-  
 tive; and  
 determining a cost of the feasible hybrid-manufactur-  
 ing plan by multiplying the atom cost and the atom  
 count for respective primitives and aggregating  
 across the set of primitives.

20. The computer system of claim 15, wherein determin- 25  
 ing the optimized hybrid-manufacturing plan further com-  
 prises:

applying a binary search to the costs to determine the  
 optimized hybrid-manufacturing plan.

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